Rules of Inference [chs. 9.1 & 9.2)

[from Irving M. Copi, *Introduction to Logic*, 8^{th} edition (NY, Macmillan, 1990)]

1. Modus Ponens (M.P.)
   p ⊃ q
   p
   ∴ q

2. Modus Tollens (M.T.)
   p ⊃ q
   ~q
   ∴ ~p

3. Hypothetical Syllogism (H.S.)
   p ⊃ q
   q ⊃ r
   ∴ p ⊃ r

4. Disjunctive Syllogism (D.S.)
   p ∨ q
   ~p
   ∴ q

5. Constructive Dilemma (C.D.)
   (p ⊃ q) • (r ⊃ s)
   p ∨ r
   ∴ q ∨ s

6. Absorption (Abs.)
   p ⊃ q
   p ⊃ (p • q)

7. Simplification (Simp.)
   p • q
   ∴ p

8. Conjunction (Conj.)
   p
   q
   ∴ p • q

9. Addition (Add.)
   p
   ∴ p ∨ q

Replacement: Any of the following logically equivalent expressions can replace each other wherever they occur.

10. De Morgan's Theorems (De M.)	$\sim(p \bullet q) \equiv (\sim p \vee \sim q)$ $\sim(p \vee q) \equiv (\sim p \bullet \sim q)$
11. Commutation (Com.)	$(p \vee q) \equiv (q \vee p)$ $(p \bullet q) \equiv (q \bullet p)$
12. Association (Assoc.)	$[p \vee (q \vee r)] \equiv [(p \vee q) \vee r]$ $[p \bullet (q \bullet r)] \equiv [(p \bullet q) \bullet r]$
13. Distribution (Dist.)	$[p \bullet (q \vee r)] \equiv [(p \bullet q) \vee (p \bullet r)]$ $[p \vee (q \bullet r)] \equiv [(p \vee q) \bullet (p \vee r]$
14. Double Negation (D.N.)	$p \equiv \sim\sim p$
15. Transposition (Trans.)	$(p \supset q) \equiv (\sim p \supset \sim q)$
16. Material Implication (Impl.)	$(p \supset q) \equiv (\sim p \vee q)$
17. Material Equivalence (Equiv.)	$(p \equiv q) \equiv [(p \supset q) \bullet (q \supset p)]$ $(p \equiv q) \equiv [(p \bullet q) \vee (\sim p \bullet \sim q)]$
18. Exportation (Exp.)	$[(p \bullet q) \supset r] \equiv [p \supset (q \supset r)]$
19. Tautology (Taut.)	$p \equiv (p \vee p)$ $p \equiv (p \bullet p)$

Quantification Rules [ch. 10]

UI: (Universal Instantiation)	$(x)(\phi x)$ $\therefore\ \phi v$	(where v is any individual symbol)
UG: (Universal Generalization)	ϕy $\therefore\ (x)(\phi x)$	(where y denotes "any arbitrarily selected individual)
EI: (Existential Instantiation)	$(\exists x)(\phi x)$ $\therefore\ \phi v$	[where v is any individual constant (other than y) having no previous occurrence in the context)
EG: (Existential Generalization)	ϕv $\therefore\ (\exists x)(\phi x)$	(where v is any individual symbol)